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Analysis of the First Lactation Curve in Holstein Cows with Different Mathematical Models

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ABSTRACT
The shape of the lactation curve of cows as well as the total or 305 day milk yield is considered as an important criterion in the livestock farms. Five different mathematical models, used in defining lactation curves were used in this study to fit first lactation curves of Holstein cattle. Total of 4472 weekly average milk yield of the first lactation of 104 cows between 2001-2008 years, was used for this aim. The models used in the study were: Wood; Morgan; Gompertz; Ali and Schaeffer and Dijkstra. The models' fit to the lactation curve has been examined and compared. Lactation curves also have been investigated according to the lactation years. The R^2, R^2_{adj}, AIC, BIC and MAPE values were used in the comparison of the models. The lowest AIC (-3.29), BIC (-3.12) and MAPE (0.55) and highest R^2 (0.99) and R^2_{adj} (0.99) values were found for the Ali and Schaeffer model. This model was followed by the Dijkstra model. As a result of the study, it was determined that the most suitable models for predicting the first lactation milk yield curves and curves features like maximum milk yield and days in milk to peak yield of Holstein cattle were Ali and Schaeffer and Dijkstra models.

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Holstein İneklerde İlk Laktasyonun Farklı Matematiksel Modellerle Analizi

ÖZET
İneklerin laktasyon eğrisinin şekli süt hayvancılığı işletmelerinde, toplam veya 305 günlük süt veriminin yanı sıra önemli bir kriter olarak kabul edilmektedir. Bu çalışmada, laktasyon eğrilerinin tanımlanmasında kullanılan beş farklı matematiksel model, Holstein sığırlarının ilk laktasyon eğrilerine uydurulmuştur. Bu amaçla, 2001-2008 yılları arasında süt verimine başlayan 104 ineğe ait 4472 adet haftalık ortalama süt verimi kullanılmıştır. Araştırmada kullanılan modeller sırası ile: Wood; Morgan; Gompertz; Ali ve Schaeffer ve Dijkstra'dır. Laktasyon eğrisine uygun modeller incelenmiş ve karşılaştırılmıştır. Laktasyon eğrileri, laktasyon yıllarına göre de incelenmiştir. Modellerin karşılaştırılmasında R^2, R^2_d, AIC, BIC ve MAPE değerleri kullanılmıştır. Sonuç olarak, Ali ve Schaeffer modeli için en düşük AIC (-3.29), BIC (-3.12) ve MAPE (0.55) ve en yüksek R^2 (0.99) ve R^2_d (0.99) değerleri bulunmuştur. Ali ve Schaeffer modelini Dijkstra modeli takip etmiştir. Çalışma sonucunda, Holstein sığırlarının ilk laktasyon süt verim eğrilerinin, maksimum süt verimi ve en yüksek verime ulaşması için gereken gün sayısı gibi özelliklerin tahmin edilmesinde en uygun modellerin Ali ve Schaeffer ve Dijkstra modelleri olduğu tespit edilmiştir.

AraştırmaMakalesi

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INTRODUCTION

The lactation curve is defined as a graphical representation of the changing of milk yield over the time after calving. With the calving, the milk yield begins and after a certain time (2-6 weeks) increases to the maximum level. The maximum level of milk production continues for a certain period of time (average a month), after which the milk yield decreases to a lower rate than the initial increase in milk yield and lactation ends when the cow becomes dry (Kaygisiz 1999; Orhan et al, 2018). The lactation curve is determined by plotting the daily milk yields according to the lactation days. The low inclination of the second part of the lactation curve in cows indicates higher persistency. Several researchers reported that cows having flat lactation curve should be preferred to cows having steep curves (Wood, 1967; Batra, 1986; Pande, 1985; Papajcsik and Bodero, 1988).

The shape of the lactation curve is considered as an important criterion in the evaluation of the total or 305-day milk yield. Lactation curves are evaluated in different application areas such as genetic evaluation, preparation of ration formulations and economic evaluation of different forms of breeding (Esenbuğa and Bilgin, 2004). In addition, the determination of lactation curve types can be used as a criterion for the culling of animals (Sherchand et al., 1995). The effects of lactation curve on milk production and economic factors are investigated by using parameters of the function of lactation curve (Grossman et al., 1986). The size of the parameters to be used in the lactation curve or, in other words, the shape of the lactation curve, is affected by genotype and the factors such as the parity, the first calving age, the service period, the drying time, the season, the management and feeding and the health status of the animal. The effects of these factors may vary from herd to herd and from year to year. It can be said that the parameters of the lactation curve calculated in a herd are specific to the herd and that the lactation curve of each herd is partially different.

To prepare a suitable breeding program, it is necessary to make a suitable herd management and production planning. For this, it is important to know the lactation curves. Mathematical models used to describe lactation curves are used to obtain features of the lactation, as well. They are generally used to estimate the total milk yield in incomplete lactations (Schaeffer et al. 1977). Although Wood model (Wood, 1967) is the best known model in this subject, many different models such as Dhanoa, Wilmink, Cobby and Le Du, Dave and Reverse Polynomial which are especially used to define lactation curve of dairy cattle have been suggested and used in researches (Morant and Gnanasakthy, 1989; Beeyer et al., 1991; Yazgan and Koncagül, 2009).

In this study, in order to develop the proper strategies by making a right selection decision, the models of Wood (Wood, 1967), Aliand Schaeffer (Ali and Schaeffer, 1987), Dijkstra (Dijkstra et al., 1997), Morgan and Gompertz (Thornley and France, 2007) were used and the model which best fits to the first lactation curves of Holstein cattlewas determined.

MATERIALS and METHODS

Material

The material of the study consisted of 4472 weekly average milk yield (AMY) records of the first lactation of 104 Holstein cows raised in a private dairy cattle farm in Konya province between 2001 and 2008. Cows were housed in a free stall barn and milked twice daily (03:00 to 06:00 and 15:00 to 18:00) in a 2 x 12side-closed milking parlour. The weekly average milk yield was calculated as the arithmetical mean of the seven days milk yield records.

Method

Lactation Curve Models

Five mathematical models were used for parameter estimation of lactation curves. These are Wood (Eq.1), Morgan (Eq.2), Gompertz (Eq.3), Aliand Schaeffer (Eq.4) and Dijkstra (Eq.5). The functions are as follows:

Wood $$Y_t = at^b e^{-ct} \quad \text{(Eq.1)}$$

Morgan $$Y_t = ab^c c \frac{t^{(c-1)}}{(t^c + b^c)^2}, \qquad c > 1 \quad \text{(Eq.2)}$$

Gompertz $$Y_t = abe^{b\frac{1-e^{-ct}}{c-ct}} \quad \text{(Eq.3)}$$

Aliand Schaeffer $$Y_t = a + b\delta_t + c\delta_t^2 + d\theta_t + g\theta_t^2 \quad \text{(Eq.4)}$$

Dijkstra $$Y_t = ae^{b\frac{1-e^{-ct}}{c-dt}} \quad \text{(Eq.5)}$$

Y_t is a milk yield of the t^{th} day of the lactation (kg), t is a days in milk (day), e is a natural logarithm base, a, b, c are the parameters of the lactation curve; a is the point where the curve intersects the y-axis; b is the rise of the curve at the start of lactation; c is the coefficient that indicates the decrease of the curve after reaching the highest level in the Eq. 1-5. In terms of Eq. 4, δ_t=t/305, θ_t=ln(305/t) and t: indicate any day from the first day of the lactation to the 305^{th} day, parameter a shows the peak milk yield, parameters d and g the increase in the curve, and the parameters b and c refer to the descent in the curve. Table 1 gives the initial milk yield – y_0 (kg), time to the maximum milk yield – t_{max} (week), maximum milk yield – y_{max} (kg) and the relative rate of decline at the point halfway between peak yield and end of lactation ($r(t_h)$) formulas for all equations (Fathi Nasri et al., 2008).

In the study, Excel (Office 2013) package was used for the preparation of the data and Nonlinear Estimation Procedure of STATISTICA 13.2 Statistical Program was used to calculate the parameters included in the models. The Levenberg-Marquardt algorithm (Levenberg, 1944; Marquardt, 1963) was used to estimate the parameters of nonlinear models.

Comparison of Lactation Curve Models

The following criteria (Eq.6-Eq.10) were used to compare lactation curve models. (Burnham and Anderson, 2002).

Table 1. Formulas for the features of lactation curves for each function

Model	yo	t_{max}	y_{max}	$r(t_h)$
Wood	0	b/c	$a(b/c)^b e^{-b}$	$2b/(t_{max}+t_f)-c$
Gompertz	ab	$c^{-1}\ln(b/c)$	$ac\,e^{(b/c-1)}$	$b e^{(-c(t_{max}+t_f)/2)} - c$
Morgan	0	$b((c-1)/(c+1))^{1/c}$	$(a/4bc)(c-1)^{(c-1)/c}(c+1)^{(c+1)/c}$	$\frac{(c-1)b^c-(c+1)((t_{max}+t_f)/2)^c}{((t_{max}+t_f)/2)(((t_{max}+t_f)/2)^c+b^c)}$
Dijsktra	a	$c^{-1}\ln(b/d)$	$a(d/b)^{d/c} e^{[(b-d)/c]}$	$b e^{(-c(t_{max}+t_f)/2)} - d$

y_o initial milk yield (kg/day); t_{max} time to peak yield (days); y_{max} maximum milk yield (kg/day); t_f, length of lactation (days); $r(t_h)$, relative rate of decline at the point halfway between peak yield and end of lactation

a) Coefficient of Determination

$$R^2 = 1 - \sum_{i=1}^{n}(y_i-\tilde{y}_i)^2 / \sum_{i=1}^{n}(y_i-\bar{y})^2 \quad \text{(Eq.6)}$$

b) Adjusted Coefficient of Determination

$$R^2_{adj} = 1-(1-R^2)\frac{n-1}{n-p} \quad \text{(Eq.7)}$$

c) Mean Absolute Percentage Error

$$MAPE = \frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{y_i-\tilde{y}_i}{y_i}\right| \quad \text{(Eq.8)}$$

d) Akaike Information Criteria

$$\mathbf{AIC} = \ln\left[\frac{1}{n}\sum_{i=1}^{n}(y_i-\tilde{y}_i)^2\right] + \frac{2p}{n-(p+1)}\ ,\left(\frac{\mathbf{n}}{\mathbf{p}} < \mathbf{40}\right) \quad \text{(Eq.9)}$$

e) Bayes Information Criteria

$$\mathbf{BIC} = \ln\left[\frac{1}{n}\sum_{i=1}^{n}(y_i-\tilde{y}_i)^2\right] + \frac{p}{n}\ln n \quad \text{(Eq.10)}$$

In the above given Eq. 6-10; n: is the number of observations, p: is the number of parameters in the model, y_i: shows milk yield per i^{th}week, $\bar{y}$: shows average milk yield, $\tilde{y}_i$: shows estimated milk yield. The highest value of the R^2 and the R^2_{adj} and the lowest value of the others were considered in determining the best model.

RESULTS and DISCUSSION

Data from animals were given in Table 2 according to years and the 43 weeks AMY of 104 cows was presented as graphic in *Fig. 1.*

Table 3 shows the prediction models and lactation curve parameters obtained from Wood, Morgan, Gompertz, Ali and Schaeffer and Dijkstra functions for the estimation of lactation curve using the AMY per week.

As seen in Table 3, Ali and Schaeffer model has five parameters, Dijkstra model has four parameters and Morgan, Wood and Gompertz models have 3 parameters. Parameters a, b and c estimated for Wood model were 20.31, 0.15 and 0.02, respectively.

In this study, a, b and c parameter values for Wood model were found to be lower than the parameter values obtained in the study conducted by İleri (2010) and Keskin and Tozluca (2004).

Table 2. 305-day milk yield per year

Years	n	%	305 day AMY** (kg)
2001	2	1.9	7726.88 a
2002	9	8.7	6527.01 ab
2003	3	2.9	7604.31 a
2004	6	5.8	7012.46 ab
2005	32	30.8	6181.35 b
2006	20	19.2	6679.61 ab
2007	24	23.1	7297.08 ab
2008	8	7.7	6751.92 ab
Total	104	100.0	

**: p<0.01

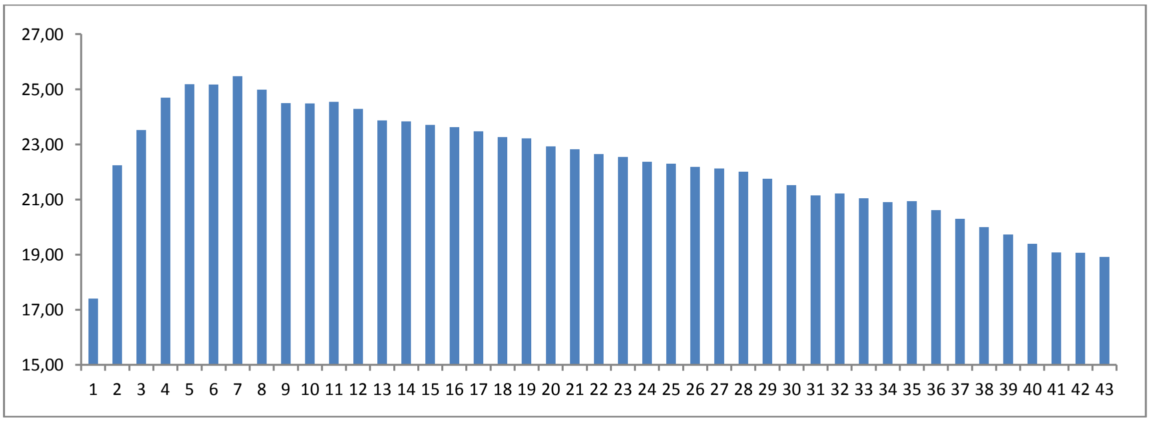

Figure 1. 305 day average milk yield per weeks

Table 3. Mathematical models and calculated parameters for lactation curve

Model	Prediction models with parameters	a	b	c	d	g
Ali and Schaeffer	Y_t=-51.92+238.01*(t/305)-648.66*(t/305)^2+32.68*ln(305/t)-3.62*(ln(305/t))^2	-51.92	238.0	-648.66	32.68	-3.62
Dijkstra	Y_t=10.84*exp(0.71*(1-exp(-0.79*t))/0.79-0.007*t)	10.84	0.71	0.79	0.007	-
Morgan	Y_t=3648.63*(106.28^1.14)*1.14*((t^(1-1.14))/(t^1.14+106.28^1.14)^2)	3648.63	106.28	1.14	-	-
Wood	Y_t=20.31*t^0.15*exp(-0.02*t)	20.31	0.15	0.02	-	-
Gompertz	Y_t=597.00*0.038*exp(0.38*(1-exp(-0.03*t))/0.03-0.03*t)	597.00	0.038	0.03	-	-

Parameters a, b and c were found to be high in the study of Orhan and Kaygısız (2002). Parameter a from the Dijsktra model was found to be less than those found in the study of Fathi Nasri et al, (2008), while it was found close to the parameter a from the study by Wasike et al (2011). Other parameters were found to be greater than the appropriate parameters. The parameters b and g for Ali and Schaeffer model were found to be higher and parameters a, c and d were found to be lower compared to the lactation curves parameters with the same sign in the study of Yazgan et al, 2013.

When we examine the lactation curve features calculated for each model, the initial milk production was over-predicted by Gompertz model and under-predicted from Dijsktra model. All models under-predicted the peak milk yield. Time to peak milk yield was over-predicted by Gompertz and Morgan models. The closest value to t_{max} valuewas obtained by Wood model. Dijsktra and Ali and Schaeffer models under-predicted the time to peak yield. These results are similar with the study by Fathi Nasri et al. (2008) (Table 4).

Evaluation criteria for the models used in the study are given in Table 5. Accordingly, the five-parameter Ali and Schaeffer model had the highest value in terms of the adjusted coefficient of determination compared to the other four and three-parameter models. According to other model comparison criteria, the smallest values were observed in Ali and Schaeffer model and in Gompertz model.

In Fig. 2, Ali and Schaeffer was the best fitted model to the observed values, followed by the Dijkstra model. When the lactation curves for AMY were evaluated by comparison criteria, it was seen that the model with the best statistics was the Ali and Schaeffer model. On the other hand, it can be said that Dijkstra model was a suitable model for the first lactation milk yield in Holstein dairy cattle by considering that there were few parameters in the applied model (Table 5 and Fig. 2).

The same data were analyzed according to the years and the evaluation criteria obtained for each model are shown in Table 6 and the plots of observed and predicted AMY are shown in Fig. 3.

The best fitted lactation curves of the animals in first lactation show similar results when the years were examined separately. According to the F test, there was a statistically significant difference between the models in terms of R^2 and R^2_{adj} ($p < 0.05$).

There was no statistically significant difference in terms of other criteria of goodness of fit. While the Gompertz model was least compatible in the t_{max} and y_{max} prediction, it gave similar results with the Ali and Schaeffer and Dijsktra models in the analysis on years (Table 6).

Table 4.y_0, t_{max},y_{max} features calculated for each model

Parameter	Models					Observed AMY
	Wood	Gompertz	Morgan	Dijsktra	Ali and Schaeffer	
y_0 (kg)	0	22.51(-5.1)	0	10.84 (6.57)	-	17.41
t_{max} (week)	7.50 (-0.50)	12.38 (-5.38)	9.72 (-2.72)	5.85 (1.15)	6 (1)	7
y_{max} (kg)	23.66 (1.81)	23.97 (1.50)	24.67 (0.80)	25.27 (0.20)	24.99 (0.48)	25.47

Table 5. Comparison criteria for each model

Model	p	R^2	R^2_{adj}	MAPE	AIC	BIC
Ali and Schaeffer	5	0.99	0.99	0.55	-3.29	-3.12
Dijkstra	4	0.98	0.98	0.90	-2.60	-2.46
Morgan	3	0.90	0.90	0.98	-0.75	-0.61
Wood	3	0.90	0.89	0.98	-0.79	-0.68
Gompertz	3	0.72	0.71	0.55	-3.29	-3.12

p: Number of parameters in the model, R^2: Coefficient of Determination, R^2_{adj}: Adjusted Coefficient of Determination, MAPE: Mean Absolute Percentage Error, AIC: Akaike Information Criteria, BIC: Bayes Information Criteria

Table 6. Model evaluation criteria applied to AMY values by years

Year	Model	p	R^2	R^2_{adj}	MAPE	AIC	BIC
2001	**Ali and Schaeffer**	**5**	**0.92**	**0.92**	**3.34**	**0.33**	**0.47**
	Dijkstra	**4**	**0.92**	**0.92**	**3.19**	**0.27**	**0.41**
	Morgan	3	0.92	0.91	3.30	0.35	0.49
	Wood	3	0.92	0.91	3.35	0.38	0.52
	Gompertz	**3**	**0.92**	**0.92**	**3.19**	**0.27**	**0.41**
2002	**Ali and Schaeffer**	**5**	**0.95**	**0.95**	**1.89**	**-1.23**	**-1.09**
	Dijkstra	4	0.73	0.72	3.86	0.51	0.65
	Morgan	3	0.90	0.90	2.48	-0.49	-0.35
	Wood	3	0.89	0.89	2.61	-0.41	-0.27
	Gompertz	**3**	**0.95**	**0.95**	**1.72**	**-1.24**	**-1.11**
2003	Ali and Schaeffer	5	0.89	0.89	2.89	0.07	0.21
	Dijkstra	**4**	**0.90**	**0.89**	**2.77**	**0.01**	**0.15**
	Morgan	3	0.88	0.87	3.21	0.17	0.31
	Wood	3	0.88	0.87	3.24	0.20	0.33
	Gompertz	**3**	**0.90**	**0.89**	**2.77**	**0.01**	**0.15**
2004	**Ali and Schaeffer**	**5**	**0.84**	**0.83**	**2.73**	**-0.24**	**-0.10**
	Dijkstra	4	0.80	0.79	3.03	-0.01	0.13
	Morgan	3	0.83	0.82	2.80	-0.17	-0.03
	Wood	3	0.83	0.82	2.79	-0.19	-0.05
	Gompertz	3	0.80	0.79	3.03	-0.01	0.13
2005	**Ali and Schaeffer**	**5**	**0.95**	**0.95**	**1.34**	**-1.73**	**-1.59**
	Dijkstra	4	0.89	0.89	2.15	-0.86	-0.72
	Morgan	3	0.84	0.83	2.67	-0.45	-0.31
	Wood	3	0.84	0.83	2.62	-0.47	-0.33
	Gompertz	3	0.87	0.86	2.40	-0.66	-0.52
2006	Ali and Schaeffer	5	0.97	0.96	1.19	-1.93	-1.79
	Dijkstra	**4**	**0.97**	**0.97**	**1.12**	**-2.04**	**-1.90**
	Morgan	3	0.84	0.83	2.42	-0.36	-0.22
	Wood	3	0.83	0.82	2.47	-0.34	-0.20
	Gompertz	**3**	**0.97**	**0.97**	**1.12**	**-2.04**	**-1.90**
2007	Ali and Schaeffer	5	0.97	0.97	1.25	-1.71	-1.57
	Dijkstra	**4**	**0.98**	**0.97**	**1.19**	**-1.91**	**-1.78**
	Morgan	3	0.83	0.82	2.68	0.02	0.16
	Wood	3	0.83	0.82	2.75	0.05	0.19
	Gompertz	**3**	**0.98**	**0.97**	**1.19**	**-1.91**	**-1.78**
2008	**Ali and Schaeffer**	**5**	**0.97**	**0.96**	**1.45**	**-1.34**	**-1.20**
	Dijkstra	4	0.94	0.94	1.65	-0.76	-0.62
	Morgan	3	0.79	0.78	2.28	0.34	0.58
	Wood	3	0.78	0.77	2.35	0.49	0.63
	Gompertz	3	0.94	0.93	1.65	-0.76	-0.62

p: Number of parameters in the model, R^2: Coefficient of Determination, R^2_{adj}: Adjusted Coefficient of Determination, MAPE: Mean Absolute Percentage Error, AIC: AkaikeInformation Criteria, BIC: Bayes Information Criteria

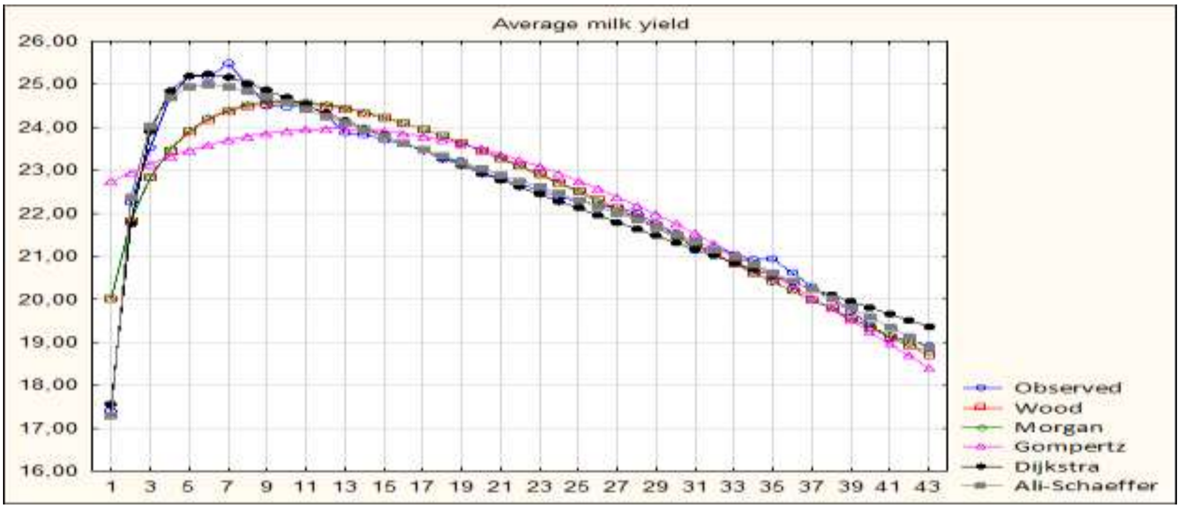

Figure 2. Plot of estimated AMY values using 5 different models and observed AMY vs weeks

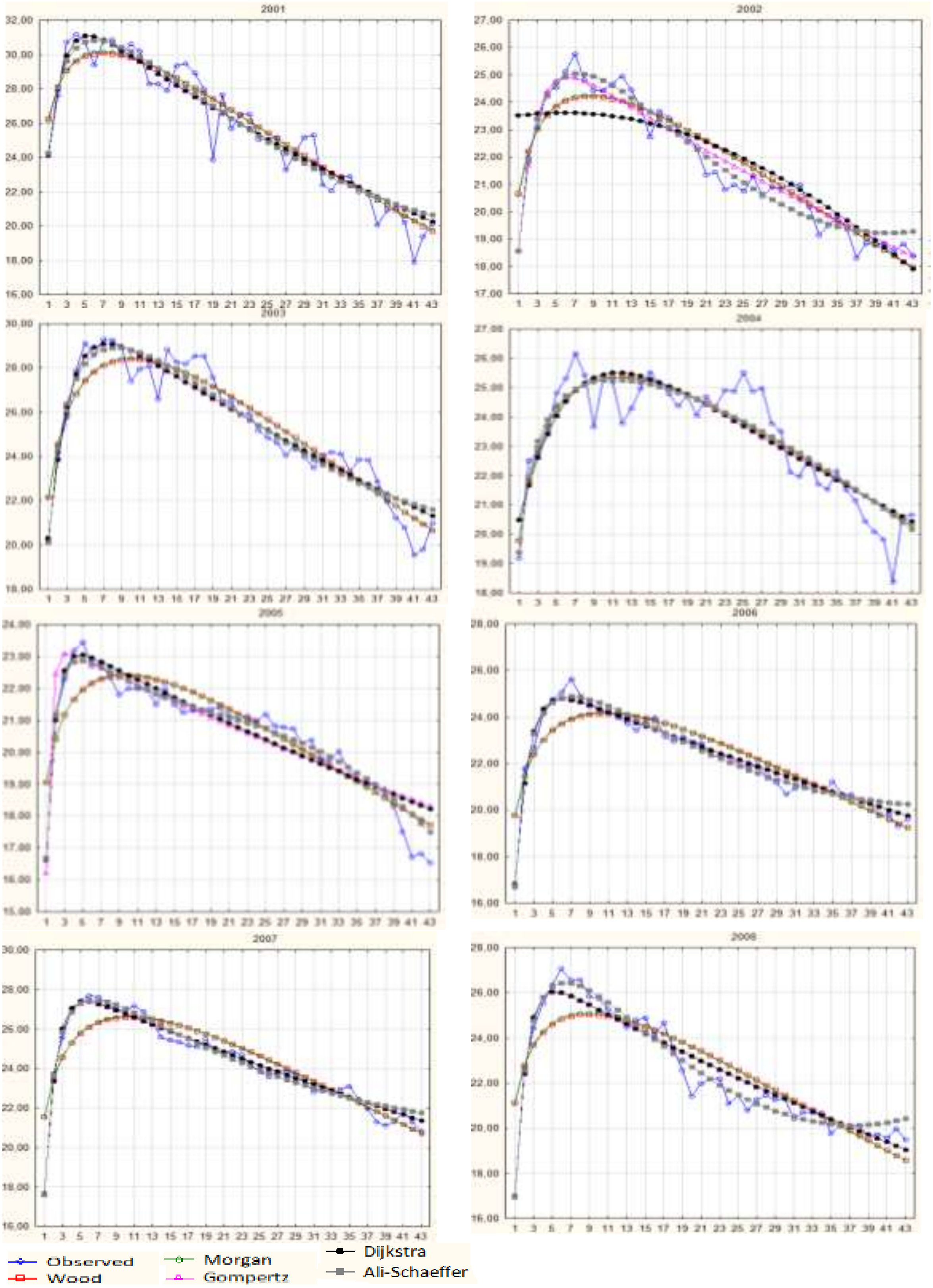

Figure 3. Plots of 305-day AMY vs weeks by years

CONCLUSION

In animal husbandry the mathematically expression of lactation allows the prediction of the milk yield that animals will give during their lactation period and during their lifetime. Foreseeing some yields requires a long time or a high cost, because it can take many years to make a correct animal breeding. Therefore, the method of estimation with mathematical models provides us time and cost benefits. When the best estimation method is determined, it will provide a time and profitable production contribution to the enterprises by making a good selection, preparing an appropriate ration by considering the lactation curve and planning the appropriate strategies to anticipate the herd's production.

In this study, the five mathematical models commonly used in dairy cattle were applied to the average milk yields of the first lactation in the sample of the Holstein cattle, and the curves were drawn and the parameters were calculated. Besides, lactation milk yield was also analyzed annually.

Wood, Ali and Schaeffer, Morgan, Dijkstra and Gompertz models analyzed in the study were evaluated by the compliance criteria such as AIC, BIC, MAPE, R^2 and R^2_{adj} and the best lactation curve fitting was observed in Ali and Schaeffer and Dijkstra models. Ali and Schaeffer and Dijksta model can be used to determine the milk yield potential and continuity in the first lactation of the animal, to estimate the amount of milk that the cow can give in future lactation, to determine the correct ration according to the lactation properties and to make the evaluation with the aim of selection. Although the average milk yield in the study gave us the information about the herd mean, it was concluded that the lactation curves should be considered on an animal basis. In future studies, it was proposed to determine the prediction models for the forecasting of 305-day milk yield by using the first four- or five-week average milk yields.

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REFERENCES

Ali TE, Schaeffer LR 1987. Accounting for covariances among test day milk yields in dairy cows. Can. J. Anim. Sci, 67: 637-644.

Batra TR 1986.Comparison of Two Mathematical Models in Fitting Lactation Curve for Pure-Line and Cross-Line Dairy Cows. Can. J. Anim. Sci., 66: 405-414.

Beever DE, Rook AJ, France J, Dhanoa MS, Gill M 1991.A Review of Empirical and Mechanistic Model of Lactational Performance by The Dairy Cow. Livest Prod Sci, 29: 115-130.

Burnham KP, Anderson DR 2002. Model Selection and Multimodel Inference – A practical Information. Theoretic Approach Second Edition, Springer-Verlag New York.

Dijkstra J, France J, Dhanoa MS, Maas JA, Hanigan MD, Rook AJ, Beever DE 1997. A Model to Describe Growth Patterns of The Mammary Gland During Pregnancy and Lactation. Journal of Dairy Science, 80: 2340–2354.

Esenbuğa N, Bilgin Ö 2004. İvesi Koyunlarının Laktasyon Eğrisinin Tahmini ve Tanımlanması için Farklı Matematik Modellerin Mukayesesi. 4.Uluslararası Zootekni Bilim Kongresi, Isparta.

Fathi Nasri MH, France J, Odongo NE, Lopez S, Bannink A, Kebreab E 2008. Modelling the Lactation Curve of Dairy Cows Using the Differentials of Growth Functions. Journal of Agricultural Science, 146: 633–641.

Grossman M, Kuck AL, Norton HW 1986. Lactation Curves of Purebred and Crossbred Dairy Cattle. Journal of Dairy Science, 69(1): 195-203.

İleri R 2010. Kazova Vasfi Diren Tarım İşletmesinde Yetiştirilen Siyah Alaca Ineklerinin Laktasyon Eğrisinin Tahmin ve Tanımlanmasıiçin Farklı Modellerin Karşılaştırılması. Yüksek Lisans Tezi, Gazi Osman Paşa Ünviversitesi Fen Bilimleri Enstitüsü. Tokat.

Kaygısız A 1999. Sarı Alaca Sığırların Laktasyon Eğrisi Özellikleri. Tr. J. of Veterinary and Animal Sciences, 23 (Eksayı 1) :15-23.

Keskin İ, Tozluca A 2004. Süt Sığırlarında Laktasyon Eğrilerinin Farklı Matematik Modellerle Belirlenmesi ve Kontrol Aralığının Tespiti. Selçuk Üniversitesi Ziraat Fakültesi Dergisi, 18(34):11-19.

Levenberg K 1944. A method for the solution of certain non-linear problems in least squares. Quarterly Journal of Applied Mathematics II (2), 164-168.

Lopez S 2008. Nonlinear Functions in Animal Nutrition. Mathematical Modelling in Animal Nutrition. / edited by J. France and E. Kebreab.p. Includes bibliographical references and index. ISBN 978-1-84593-354-8 (alk. paper)

Marquardt DW 1963. An algorithm for least-squares estimation of non-linear parameters. Journal of the Society of Industrial and Applied Mathematics 11 (2), 431-441.

Morant SV, Gnanasakthy A 1989. A New Approach to The Mathematical Formulation of Lactation Curves.Animal Production,49: 151-162.

Orhan H, Kaygısız A 2002. Siyah Alaca Sığırlarda Farklı Laktasyon Eğrisi Modellerinin Karşılaştırılması. Hayvansal Üretim, 43: 94-99.

Orhan H, Teke ÇE, Karcı Z 2018. Laktasyon Eğrileri Modellemesinde Çok Değişkenli Uyarlanabilir Regresyon Eğrileri (Mars) Yönteminin Uygulanması. KSU J Agric Nat 21(3) : 363-373.

Pande AM 1985. Studies on the Lactation Curve and Components of Lactation Curve in Gaolao and itsCrosses with Exotic Breeds. Animal Breeding Abstract, 53: 26-49.

Papajcsik IA, Bodero J 1988. Modeling Lactation Curves of Friesian Cows in A Subtropical Climate. Animal Production, 47: 201-207.

Schaeffer LR, Minder CE, Mc Millan F, Burnside EB 1977. Nonlinear Techniques for Predicting 305 – Day Lactation Production of Holstein and Jersey. Journal of Dairy Science 60: 1636–1644.

Sherchand L, McNew RW, Kellogg DW, Johnson ZB 1995. Selection of A Mathematical Model to

Generate Lactation Curves Using Daily Milk Yields of Holstein Cows. Journal of Dairy Science, 78: 2507-2513.

Thornley JHM, France J 2007. Mathematical Models in Agriculture: Quantitative Methods Forthe Plant, Animal and Ecological Sciences, 2nd ed. CAB International, Wallingford, UK.

Wasike CB, Kahi AK, Peters KJ 2011. Modelling of Lactation Curves of Dairy Cows Based on Monthly Test Day Milk Yield Records under Inconsistent Milk Recording Scenarios. Animal, 5(11): 1780-1790.

Wood PDP 1967. Algebraic Model of Lactation Curve in Cattle. Nature 218:164-165.

Yazgan K, Koncagül S 2009. Laktasyon Eğrilerinin Tanımlanmasında Doğrusal ve Kübik Spline ile Legendre Polinomial Fonksiyonlarının Kullanımı. – VI Zootekni Bilim Kongresi, Erzurum.

Yazgan K, Koncagül S, Cedden F 2013. Siyah Alaca Irkı Süt İneklerinde Ali-Schaeffer Modeli Kullanılarak Tanımlanmış Farklı Laktasyon Eğrisi Biçimlerinin Belirlenmesi. Harran Üniv Vet FakDerg, 2(2): 94-101.